Probability

1. Imagine that you have a box with 2 blue marbles and 5 red marbles in it.

   (a) What is the chance of drawing a blue marble?
   A: $\frac{2}{7}$

   (b) What is the chance of drawing a red and a blue marble if you sample with replacement?
   A: $2!\frac{5}{7}\frac{2}{7} = \frac{20}{49}$

   (c) Imagine drawing all the marbles from the box. How many different orderings of drawn marbles are possible?
   A: $\binom{7}{2} = 21$

   (d) What is the chance of the most probable ordering?
   A: $\frac{5}{7}\frac{4}{6}\frac{3}{5}\frac{2}{4}\frac{2}{3} = \frac{2}{21}$

   (e) Imagine I now have two boxes; the first is the same as above, but the second has 4 blue marbles and 3 red marbles. I draw from one of the boxes at random, and show you that I picked a blue marble. What is the chance that I drew the marble from the first box?
   A: $P(\text{Box 1}|\text{Blue}) = \frac{P(\text{Blue}|\text{Box 1})P(\text{Box 1})}{P(\text{Blue})} = \frac{\frac{2}{7}\frac{1}{2}}{\frac{1}{2}(\frac{2}{7}+\frac{4}{7})} = \frac{1}{3}$

2. Imagine you have a six-sided die and a ten-sided die.

   (a) What is the expected value of rolling the six-sided die?
   A: 3.5

   (b) What is the expected value of rolling both dice?
   A: $3.5 + 5.5 = 9.0$

   (c) What is the variance of the sum of both dice?
   A: $\frac{35}{12} + 8.25 = 11 + \frac{1}{6}$

(d) What is the chance of the sum of the two dice being 4?
A: $3\frac{1}{6}\frac{1}{10} = \frac{1}{20}$

(e) Imagine that I tell you I have rolled a 4 with one of the dice. It is twice as likely that for any given roll I will use the ten-sided die. What is the chance that I rolled the ten-sided die for this roll?
A: $P(10S|4) = \frac{P(4|10S)P(10S)}{P(4)} = \frac{\frac{1}{10}\frac{2}{3}}{\frac{1}{3}(2\frac{1}{10}+\frac{1}{6})} = \frac{6}{11}$

3. Here's a sample from a distribution:
$\{-2, -1, 3, 3, 5, 6, 8, 10, 11\}$

(a) What is the mean of the sample?
A: $\frac{43}{9} = 4.7778$

(b) What is the median?
A: 5

(c) What is the mode?
A: 3

(d) What is the standard deviation?
A: $\sqrt{\frac{184}{9}} = 4.522$

4. Consider a continuous normal distribution with $\mu = 0$ and $\sigma = 3$.

(a) What is the variance?
A: 9

(b) What is the median?
A: 0

(c) What is the probability of sampling from the distribution and getting 0?
A: 0

(d) What is the probability of getting less than -1?
A: .3707